Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Summer Examination-2018

Subject Name : Engineering Mathematics – I

Subject Code : 4TE01EMT1 **Branch: B.Tech (All)**

Semester : 1 **Date : 21/03/2018** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) The points representing the complex number z for which $\arg\left(\frac{z-2}{z+2}\right)=\frac{\pi}{3}$ lie on

(A) a circle (B) a straight line (C) an ellipse (D) a parabola

b) If $\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)^{50}=3^{25}\left(x-iy\right)$ where $x,\ y$ are real, then the ordered pair $\left(x,y\right)$ is given by

(A) $\left(0,\ 3\right)$ (B) $\left(\frac{1}{2},\ \frac{\sqrt{3}}{2}\right)$ (C) $\left(-3,\ 0\right)$ (D) $\left(0,\ -3\right)$

c) If $f\left(x\right)=\frac{e^{x}-e^{-x}}{2}$ is continuous at $x=0$, then the value of $f\left(0\right)$ must be

(A) 0 (B) 1 (C) 2 (D) 3

d) $\lim_{x\to\infty}\frac{x^2}{e^x}=$ _____

(A) -1 (B) 0 (C) 1 (D) none of these

e) The infinite series $1+r+r^2+.....+r^{n-1}$ is divergent if

(A) $|r|<1$ (B) $|r|>1$ (C) $r\geq 1$ (D) $r=-1$

f) The series $1+\frac{1}{2}+\frac{1}{3}+.....+\frac{1}{n}+.....\infty$ is

(A) convergent (B) divergent (C) absolutely convergent (D) none of these

g) If the two tangents at the point are imaginary, the double point is called

(A) a node (B) a cusp (C) a conjugate point (D) none of these

h) The curve $x^3+y^3=3axy$ represent

(A) Cissoid of Diocle (B) Witch of Agnesi (C) Strophoid
(D) Folium of Descartes

i) The series $x+\frac{x^3}{3!}+\frac{x^5}{5!}+.....$ represent expansion of

(A) $\sin x$ (B) $\cos x$ (C) $\sinh x$ (D) $\cosh x$

j) If $y=\cos^{-1}x$, then x equal to

(A) $1-\frac{y^2}{2!}+\frac{y^4}{4!}-.....$ (B) $y-\frac{y^3}{3!}+\frac{y^5}{5!}-.....$ (C) $1+y+\frac{y^2}{2!}+\frac{y^3}{3!}$

(D) none of these

k) Conditions for $f(x,y)$ to be maximum are

(A) $f_x=0=f_y,\ rt<s^2,\ r<0$ (B) $f_x=0=f_y,\ rt>s^2,\ r<0$

(C) $f_x=0=f_y,\ rt>s^2,\ r>0$ (D) $f_x=0=f_y,\ rt=s^2,\ r>0$

l) If $u=y^x$, then $\frac{\partial u}{\partial x}$ is

(A) xy^{x-1} (B) 0 (C) $y^x\log x$ (D) none of these

m) $\frac{\partial(u,v,w)}{\partial(x,y,z)}\cdot\frac{\partial(x,y,z)}{\partial(u,v,w)}=$ ______

(A) 2 (B) 1 (C) 0 (D) none of these

n) If $f(x,y)=0$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{\partial f/\partial x}{\partial f/\partial y}$ (B) $\frac{\partial f/\partial y}{\partial f/\partial x}$ (C) $-\frac{\partial f/\partial y}{\partial f/\partial x}$ (D) $-\frac{\partial f/\partial x}{\partial f/\partial y}$

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Prove that $(a+ib)^{\frac{m}{n}}+(a-ib)^{\frac{m}{n}}=2\left(a^2+b^2\right)^{\frac{m}{2n}}\cos\left(\frac{m}{n}\tan^{-1}\frac{b}{a}\right)$. **(5)**

b) Using Sandwich theorem show that **(5)**

(i) $\lim_{x\to 0}x^2\sin\left(\frac{1}{x}\right)=0$ (ii) $\lim_{x\to 0}x^2\cos 20\pi x=0$

c) Evaluate: $\lim_{x\to 0}\frac{e^x+e^{-x}-x^2-2}{\sin^2 x-x^2}$ **(4)**

Q-3 **Attempt all questions** **(14)**

a) Using De Moivre's theorem prove that $\sin 5\theta=5\sin\theta-20\sin^3\theta+16\sin^5\theta$. **(5)**

b) Prove that $\lim_{x\to 1}\left(\frac{x}{x-1}-\frac{1}{\log x}\right)=\frac{1}{2}$. **(5)**

c) Find the roots of the equation $x^6-i=0$. **(4)**

Q-4 **Attempt all questions** **(14)**

a) Expand $f(x)=\frac{e^x}{e^x+1}$ in powers of x up to x^3 by Maclaurin's series. **(5)**

b) Prove that $\tan^{-1}x=x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+.....$ **(5)**

c) Test for convergence the series $\sum \sin\left(\frac{1}{n}\right)$. **(4)**

Q-5 **Attempt all questions** **(14)**

a) Show that $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is (i) convergent if $p > 1$ and (ii) divergent if $p \le 1$. **(5)**

b) Test for convergence the series $\frac{1}{1\cdot 2\cdot 3} + \frac{x}{4\cdot 5\cdot 6} + \frac{x^2}{7\cdot 8\cdot 9} + ...$ **(5)**

c) Calculate approximate value of $\sqrt{9.12}$ by using Taylor's theorem. **(4)**

Q-6 **Attempt all questions** **(14)**

a) If $u = \sec^{-1}\left(\frac{x^2 + y^2}{x - y}\right)$ then prove that **(5)**

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -\cot u\left(\cot^2 u + 2\right).$$

b) If $u = \log\left(x^3 + y^3 + z^3 - 3xyz\right)$ then verify $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x + y + z)^2}$. **(5)**

c) Find the asymptotes of the curve $x^2 y^2 - x^2 y - 5xy^2 + x + y + 5 = 0$. **(4)**

Q-7 **Attempt all questions** **(14)**

a) Trace the curve $r^2 = a^2 \cos 2\theta$. **(5)**

b) Discuss the maxima and minima of $xy + 27\left(\frac{1}{x} + \frac{1}{y}\right)$. **(5)**

c) If $u = \tan^{-1}\left(\frac{y}{x}\right)$ then verify that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$. **(4)**

Q-8 **Attempt all questions** **(14)**

a) Trace the curve $y^2(2a - x) = x^3$. **(5)**

b) The power consumed in an electric resistor is given by $P = \frac{E^2}{R}$ (in watts). If $E = 200$ volts and $R = 8$ ohms, by how much does the power change if E is decreased by 5 volts and R is decreased by 0,20 ohms? **(5)**

c) If $u = 2xy,\ v = x^2 - y^2$ and $x = r\cos\theta,\ y = r\sin\theta$ then evaluate $\frac{\partial(u,\ v)}{\partial(r,\ \theta)}$. **(4)**